Divulgaciones Matemáticas Vol. 19 No. 2 (2018), pp. 1–13

# On contra $e^*\theta$-continuous functions

*En funciones contra $e^*\theta$-continuas*

Burcu Sünbül Ayhan (brcyhn@gmail.com)
Murad Özkoç (murad.ozkoc@mu.edu.tr)
Muğla Sıtkı Koçman University
Faculty of Science Department of Mathematics
48000 Menteşe-Muğla/TURKEY

**Abstract**

The main goal of this paper is to introduce and study a new type of contra continuity called contra $e^*\theta$-continuity. Also, we obtain fundamental properties and several characterizations of contra $e^*\theta$-continuous functions via $e^*$-$\theta$-closed sets which are defined by Farhan and Yang [11]. Moreover, we investigate the relationships between contra $e^*\theta$-continuous functions and other related generalized forms of contra continuity.

**Key words and phrases:** $e^*$-$\theta$-open set, $e^*$-$\theta$-closed set, contra $e^*\theta$-continuity, $e^*\theta$-continuity, contra $e^*\theta$-closed graph.

**Resumen**

El objetivo principal de este documento es presentar y estudiar un nuevo tipo de contra continuidad llamada contra $e^*\theta$-continuidad. Además, obtenemos propiedades fundamentales y varias caracterizaciones de funciones contra $e^*\theta$-continuas a través de conjuntos $e^*$-$\theta$ cerrados que están definidos por Farhan y Yang [11]. Además, investigamos las relaciones entre las funciones contra continuas y otras formas generalizadas relacionadas de $e^*\theta$-continuidad de contra.

**Palabras y frases clave:** $e^*$-$\theta$-conjunto abierto, $e^*$-$\theta$-conjunto cerrado, contra $e^*\theta$-continuidad, $e^*\theta$-continuidad, contra $e^*\theta$-gráfico cerrado.

## 1 Introduction

In 1996, the concept of contra continuity [6], which is stronger than contra $\alpha$-continuity [12], contra precontinuity [13], contra semicontinuity [7], contra $b$-continuity [17], contra $\beta$-continuity [5], is defined by Dontchev. Many results have been obtained related to the notions mentioned above recently. In this paper, we define and study the notion of contra $e^*\theta$-continuity which is stronger than contra $e^*$-continuity [10] and weaker than contra $\beta\theta$-continuity [4]. Also, we obtain several characterizations of contra $e^*\theta$-continuous functions and investigate their some fundamental properties. Moreover, we investigate the relationships between contra $e^*\theta$-continuous functions and seperation axioms and contra $e^*\theta$-closedness of graphs of functions.

Received 28/05/2018. Revised 15/08/2019. Accepted 25/11/2018.
MSC (2000): Primary 54C08, 54C10; Secondary 54C05.
Corresponding author: Murad Özkoç

## 2 Preliminaries

Throughout this present paper, $X$ and $Y$ represent topological spaces. For a subset $A$ of a space $X$, $cl(A)$ and $int(A)$ denote the closure of $A$ and the interior of $A$, respectively. The family of all closed (resp. open, clopen) sets of $X$ is denoted $C(X)$(resp. $O(X), CO(X)$). A subset $A$ is said to be regular open [23] (resp. regular closed [23]) if $A = int(cl(A))$ (resp. $A = cl(int(A))$). A point $x \in X$ is said to be $\delta$-cluster point [24] of $A$ if $int(cl(U)) \cap A \neq \emptyset$ for each open neighbourhood $U$ of $x$. The set of all $\delta$-cluster points of $A$ is called the $\delta$-closure [24] of $A$ and is denoted by $cl_\delta(A)$. If $A = cl_\delta(A)$, then $A$ is called $\delta$-closed [24], and the complement of a $\delta$-closed set is called $\delta$-open [24]. The set $\{x|(U \in O(X,x))(int(cl(U)) \subseteq A)\}$ is called the $\delta$-interior of $A$ and is denoted by $int_\delta(A)$.

A subset $A$ is called $\alpha$-open [18] (resp. semiopen [14], preopen [15], $b$-open [2], $\beta$-open [1], $e$-open [8], $e^*$-open [9]) if $A \subseteq int(cl(int(A)))$ (resp. $A \subseteq cl(int(A))$, $A \subseteq int(cl(A))$, $A \subseteq int(cl_\delta(A))$, $A \subseteq cl(int(A)) \cup int(cl(A))$, $A \subseteq cl(int(cl(A)))$, $A \subseteq cl(int_\delta(A)) \cup int(cl_\delta(A))$, $A \subseteq cl(int(cl_\delta(A)))$). The complement of an $\alpha$-open (resp. semiopen, preopen, $b$-open, $\beta$-open, $e$-open, $e^*$-open) set is called $\alpha$-closed [18] (resp. semiclosed [14], preclosed [15], $b$-closed [2], $\beta$-open [1], $e$-closed [8], $e^*$-closed [9]). The intersection of all $e^*$-closed (resp. semi-closed, pre-closed) sets of $X$ containing $A$ is called the $e^*$-closure [9] (resp. semi-closure [14], pre-closure [15]) of $A$ and is denoted by $e^*\text{-}cl(A)$ (resp. $scl(A)$, $pcl(A)$). The union of all $e^*$-open (resp. semiopen, preopen) sets of $X$ contained in $A$ is called the $e^*$-interior [9] (resp. semi-interior [14], pre-interior [15]) of $A$ and is denoted by $e^*\text{-}int(A)$ (resp. $sint(A)$, $pint(A)$).

The union of all $e^*$-open sets of $X$ contained in $A$ is called the $e^*$-interior [9] of $A$ and is denoted by $e^*\text{-}int(A)$. A subset $A$ is said to be $e^*$-regular [11] if it is $e^*$-open and $e^*$-closed. The family of all $e^*$-regular subsets of $X$ is denoted by $e^*R(X)$.

A point $x$ of $X$ is called an $e^*$-$\theta$-cluster ($\beta$-$\theta$-cluster) point of $A$ if $e^*\text{-}cl(U) \cap A \neq \emptyset$ for every $e^*$-open (resp. $\beta$-open) set $U$ containing $x$. The set of all $e^*$-$\theta$-cluster ($\beta$-$\theta$-cluster) points of $A$ is called the $e^*$-$\theta$-closure [11] ($\beta$-$\theta$-closure [19]) of $A$ and is denoted by $e^*\text{-}cl_\theta(A)$ ($\beta\text{-}cl_\theta(A)$). A subset $A$ is said to be $e^*$-$\theta$-closed [11] ($\beta$-$\theta$-closed [19]) if $A = e^*\text{-}cl_\theta(A)$ ($A = \beta\text{-}cl_\theta(A)$). The complement of an $e^*$-$\theta$-closed ($\beta$-$\theta$-closed) set is called an $e^*$-$\theta$-open [11] ($\beta$-$\theta$-open [19]) set. A point $x$ of $X$ is said to be an $e^*$-$\theta$-interior [11] ($\beta$-$\theta$-interior [19]) point of a subset $A$, denoted by $e^*\text{-}int_\theta(A)$ ($\beta\text{-}int_\theta(A)$), if there exists an $e^*$-open ($\beta$-open) set $U$ of $X$ containing $x$ such that $e^*\text{-}cl(U) \subseteq A$ ($\beta\text{-}cl(U) \subseteq A$). Also it is noted in [11] that

$$e^*\text{-regular} \Rightarrow e^*\text{-}\theta\text{-open} \Rightarrow e^*\text{-open}.$$

The family of all open (resp. closed, $e^*$-$\theta$-open, $e^*$-$\theta$-closed, $e^*$-open, $e^*$-closed, regular open, regular closed, $\delta$-open, $\delta$-closed, semiopen, semiclosed, preopen, preclosed) subsets of $X$ is denoted by $O(X)$ (resp. $C(X)$, $e^*\theta O(X)$, $e^*\theta C(X)$, $e^*O(X)$, $e^*C(X)$, $RO(X)$, $RC(X)$, $\delta O(X)$, $\delta C(X)$, $SO(X)$, $SC(X)$, $PO(X)$, $PC(X)$). The family of all open (resp. closed, $e^*$-$\theta$-open, $e^*$-$\theta$-closed, $e^*$-open, $e^*$-closed, regular open, regular closed, $\delta$-open, $\delta$-closed, semiopen, semiclosed, preopen, preclosed) sets of $X$ containing a point $x$ of $X$ is denoted by $O(X,x)$ (resp. $C(X,x)$, $e^*\theta O(X,x)$, $e^*\theta C(X,x)$, $e^*O(X,x)$, $e^*C(X,x)$, $RO(X,x)$, $RC(X,x)$, $\delta O(X,x)$, $\delta C(X,x)$, $SO(X,x)$, $SC(X,x)$, $PO(X,x)$, $PC(X,x)$).

We shall use the well-known accepted language almost in the whole of the proofs of the theorems in this article. The following basic properties of $e^*$-closure and $e^*$-interior are useful in the sequel:

**Lemma 2.1.** *[9] Let $A$ be a subset of a space $X$, then the following hold:*

(1) $e^*\text{-}cl(X \setminus A) = X \setminus e^*\text{-}int(A)$.

(2) $x \in e^*\text{-}cl(A)$ *if and only if* $A \cap U \neq \emptyset$ *for every* $U \in e^*O(X, x)$.

(3) $A$ *is* $e^*C(X)$ *if and only if* $A = e^*\text{-}cl(A)$.

(4) $e^*\text{-}cl(A) \in e^*C(X)$.

(5) $e^*\text{-}int(A) = A \cap cl(int(cl_\delta(A)))$.

**Lemma 2.2.** *[11] For the* $e^*\theta$*-closure of a subset* $A$ *of a topological space* $X$, *the following properties are hold:*

(1) $A \subseteq e^*\text{-}cl(A) \subseteq e^*\text{-}cl_\theta(A)$.

(2) *If* $A \in e^*\theta O(X)$, *then* $e^*\text{-}cl_\theta(A) = e^*\text{-}cl(A)$.

(3) *If* $A \subseteq B$, *then* $e^*\text{-}cl_\theta(A) \subseteq e^*\text{-}cl_\theta(B)$.

(4) $e^*\text{-}cl_\theta(A) \in e^*\theta C(X)$ *and* $e^*\text{-}cl_\theta(e^*\text{-}cl_\theta(A)) = e^*\text{-}\ cl_\theta(A)$.

(5) *If* $A_\alpha \in e^*\theta C(X)$ *for each* $\alpha \in \Lambda$, *then* $\cap\{A_\alpha | \alpha \in \Lambda\} \in e^*\theta C(X)$.

(6) *If* $A_\alpha \in e^*\theta O(X)$ *for each* $\alpha \in \Lambda$, *then* $\bigcup\{A_\alpha | \alpha \in \Lambda\} \in e^*\theta O(X)$.

(7) $e^*\text{-}cl_\theta(X \setminus A) = X \setminus e^*\text{-}int_\theta(A)$.

(8) $e^*\text{-}cl_\theta(A) = \cap\{U | (A \subseteq U)(U \in e^*\theta C(X))\}$.

(9) $A \in e^*O(X)$, *then* $e^*\text{-}cl_\theta(A) \in e^*R(X)$.

(10) $A \in e^*R(X)$ *if and only if* $A \in e^*\theta O(X) \cap e^*\theta C(X)$.

**Lemma 2.3.** *Let* $A$ *be a subset of a topological space* $X$ *and* $x \in X$. *The point* $x$ *of* $X$ *is an* $e^*$*-*$\theta$*-cluster point of* $A$ *if and only if* $U \cap A \neq \emptyset$ *for all* $e^*$*-*$\theta$*-open* $U$ *containing* $x$.

*Proof.* Let $x \notin e^*\text{-}cl_\theta(A)$.

$$\begin{array}{rcl} x \notin e^*\text{-}cl_\theta(A) & \Leftrightarrow & (\exists U \in e^*\theta C(X))(A \subseteq U)(x \notin U) \\ & \Leftrightarrow & (\exists \setminus U \in e^*\theta O(X))(\setminus U \subseteq \setminus A)(x \in \setminus U) \\ & \Leftrightarrow & (\exists V := \setminus U \in e^*\theta O(X, x))(V \subseteq \setminus A) \\ & \Leftrightarrow & (\exists V \in e^*\theta O(X, x))(V \cap A = \emptyset) \\ & \Leftrightarrow & x \notin \{x | (\forall U \in e^*\theta O(X, x))(U \cap A = \emptyset)\}. \quad \square \end{array}$$

**Definition 2.1.** A function $f : X \to Y$ is said to be contra continuous [6] (resp. contra $\alpha$-continuous [12], contra precontinuous [13], contra semicontinuous [7], contra $b$-continuous [17], contra $\beta$-continuous [5], contra $\beta\theta$-continuous [4], contra $e^*$-continuous [10]) if $f^{-1}[V]$ is closed (resp. $\alpha$-closed, preclosed, semiclosed, $b$-closed, $\beta$-closed, $\beta$-$\theta$-closed, $e^*$-closed) in $X$ for every open set $V$ in $Y$.

**Definition 2.2.** Let $A$ be a subset of a space $X$. The intersection of all open sets in $X$ containing $A$ is called the kernel of $A$ [16] and is denoted by $ker(A)$.

**Lemma 2.4.** *[16] The following properties hold for subsets $A$ and $B$ of a space $X$.*

*(1)* $x \in ker(A)$ *if and only if* $A \cap F \neq \emptyset$ *for any* $F \in C(X, x)$.

*(2)* $A \subseteq ker(A)$.

*(3) If $A$ is open in $X$, then $A = ker(A)$.*

*(4) If $A \subseteq B$, then $ker(A) \subseteq ker(B)$.*

## 3 Contra $e^*\theta$-continuous functions

**Definition 3.1.** A function $f : X \to Y$ is said to be contra $e^*\theta$-continuous (briefly c.$e^*\theta$.c.) if $f^{-1}[V]$ is $e^*$-$\theta$-closed in $X$ for every open set $V$ of $Y$.

**Theorem 3.1.** *For a function $f : X \to Y$, the following properties are equivalent:*

*(1) $f$ is contra $e^*\theta$-continuous;*

*(2) The inverse image of every closed set of $Y$ is $e^*$-$\theta$-open in $X$;*

*(3) For each point $x \in X$ and each and each $V \in C(Y, f(x))$, there exists $U \in e^*\theta O(X, x)$ such that $f[U] \subseteq V$;*

*(4) $f[e^*\text{-}cl_\theta(A)] \subseteq ker(f[A])$ for every subset $A$ of $X$;*

*(5) $e^*\text{-}cl_\theta(f^{-1}[B])] \subseteq f^{-1}[ker(B)]$ for every subset $B$ of $Y$.*

*Proof.*
(1) ⇒ (2) : Let $V \in C(Y)$.

$$\left.\begin{array}{r} V \in C(Y) \Rightarrow \backslash V \in O(Y) \\ (1) \end{array}\right\} \Rightarrow \backslash f^{-1}[V] = f^{-1}[\backslash V] \in e^*\theta C(X) \Rightarrow f^{-1}[V] \in e^*\theta O(X)$$

(2) ⇒ (3) : Let $x \in X$ and $V \in C(Y, f(x))$.

$$\left.\begin{array}{r} (x \in X)(V \in C(Y, f(x))) \\ (2) \end{array}\right\} \Rightarrow \left.\begin{array}{r} f^{-1}[V] \in e^*\theta O(X, x) \\ U := f^{-1}[V] \end{array}\right\} \Rightarrow (U \in e^*\theta O(X, x))(f[U] \subseteq V).$$

(3) ⇒ (4) : Let $A \subseteq X$ and $x \notin f^{-1}[ker(f[A])]$.

$$\left.\begin{array}{r} x \notin f^{-1}[ker(f[A])] \Rightarrow f(x) \notin ker(f[A]) \Rightarrow (\exists F \in C(Y, f(x)))(F \cap f[A] = \emptyset) \\ (3) \end{array}\right\} \Rightarrow$$

$$\begin{array}{l} \Rightarrow (\exists U \in e^*\theta O(X, x))(f[U] \subseteq F)(F \cap f[A] = \emptyset) \\ \Rightarrow (\exists U \in e^*\theta O(X, x))(f[U \cap A] \subseteq f[U] \cap f[A] = \emptyset) \\ \Rightarrow (\exists U \in e^*\theta O(X, x))(U \cap A = \emptyset) \\ \Rightarrow x \notin e^*\text{-}cl_\theta(A). \end{array}$$

$(4) \Rightarrow (5)$ : Let $B \subseteq Y$.

$$\left.\begin{array}{r} B \subseteq Y \Rightarrow f^{-1}[B] \subseteq X \\ (4) \end{array}\right\} \Rightarrow f[e^*\text{-}cl_\theta(f^{-1}[B])] \subseteq ker(f\left[f^{-1}[B]\right]) \subseteq ker(B) \Rightarrow$$

$$\Rightarrow e^*\text{-}cl_\theta(f^{-1}[B]) \subseteq f^{-1}[ker(B)].$$

$(5) \Rightarrow (1)$ : Let $V \in O(Y)$.

$$\left.\begin{array}{r} V \in O(Y) \\ (5) \end{array}\right\} \Rightarrow e^*\text{-}cl_\theta(f^{-1}[V]) \subseteq f^{-1}[ker(V)] = f^{-1}[V] \Rightarrow f^{-1}[V] \in e^*\theta C(X). \quad \square$$

*Remark* 3.1. From Definitions 3.1 and 2.1, we have the following diagram. None of these implications is reversible as shown by the following example:

$$\begin{array}{ccccccccc}
 & & \text{c.}p\text{.c.} & & & & \text{c.}\beta\text{.c.} & \leftarrow & \text{c.}\beta\theta\text{.c.} \\
 & \nearrow & & \searrow & & \nearrow & & & \\
\text{c.}\alpha\text{.c.} & \leftarrow & \text{c.c.} & & \text{c.}b\text{.c.} & & \downarrow & & \downarrow \\
 & \searrow & & \nearrow & & \searrow & & & \\
 & & \text{c.s.c.} & & & & \text{c.}e^*\text{.c.} & \leftarrow & \text{c.}e^*\theta\text{.c.}
\end{array}$$

*Notation* 3.1. c.c.=contra continuity, c.$\alpha$.c.=contra $\alpha$-continuity, c.$p$.c.=contra precontinuity, c.$s$.c.=contra semicontinuity, c.$b$.c.=contra b-continuity, c.$\beta$.c.= contra $\beta$-continuity, c.$e^*$.c.=contra $e^*$-continuity, c.$\beta\theta$.c.=contra $\beta\theta$-continuity, c.$e^*\theta$.c.=contra $e^*\theta$-continuity.

**Example 3.1.** *Let* $X = \{a, b, c, d\}$ *and* $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}\}$. *It is not difficult to see that*

$$e^*\theta O(X) = e^*O(X) = 2^X \setminus \{\{d\}\} \quad \text{and} \quad \beta\theta C(X) = \{\emptyset, X, \{a, c, d\}, \{b, d\}, \{a, c\}, \{c\}, \{d\}\}.$$

*Define the function* $f : X \to X$ *by* $f = \{(a, c), (b, b), (c, a), (d, b)\}$. *Then* $f$ *is contra* $e^*\theta$*-continuous but it is not contra* $\beta\theta$*-continuous.*

Other examples can be found related articles.

**Definition 3.2.** A function $f : X \to Y$ is said to be:

a) $e^*\theta$-semiopen if $f[U] \in SO(Y)$ for every $e^*$-$\theta$-open set $U$ of $X$.

b) contra $I(e^*\theta)$-continuous if for each $x$ in $X$ and each $V \in C(Y, f(x))$, there exists $U \in e^*\theta O(X, x)$ such that $int(f[U]) \subseteq V$.

c) $e^*\theta$-continuous [11] if $f^{-1}[V]$ is $e^*\theta$-closed in $X$ for every closed set $V$ of $Y$.

d) $e^*$-continuous [9] if $f^{-1}[V]$ is $e^*$-closed in $X$ for every closed set $V$ of $Y$.

**Theorem 3.2.** *Let* $f : X \to Y$ *be a function. If* $f$ *is contra* $I(e^*\theta)$*-continuous and* $e^*\theta$*-semiopen, then* $f$ *is contra* $e^*\theta$*-continuous.*

*Proof.* Let $x \in X$ and $V \in C(Y, f(x))$.

$$\left.\begin{array}{r} (x \in X)(V \in C(Y, f(x))) \\ f \text{ is contra } I(e^*\theta)\text{-continuous} \end{array}\right\} \Rightarrow \left.\begin{array}{r} (\exists U \in e^*\theta O(X, x))(int(f[U]) \subseteq V = cl(V)) \\ f \text{ is } e^*\theta\text{-semiopen} \end{array}\right\} \Rightarrow$$

$$\Rightarrow (\exists U \in e^*\theta O(X, x))(f[U] \in SO(Y))(int(f[U]) \subseteq V = cl(V))$$

$$\Rightarrow (\exists U \in e^*\theta O(X, x))(f[U] \subseteq cl(int(f[U])) \subseteq V). \quad \square$$

**Theorem 3.3.** *Let $f : X \to Y$ be a function. If $f$ is contra $e^*\theta$-continuous and $Y$ is regular, then $f$ is $e^*\theta$-continuous.*

*Proof.* Let $x \in X$ and $V \in O(Y, f(x))$.

$$\left.\begin{array}{r}(x \in X)(V \in O(Y, f(x)))\\ Y \text{ is regular}\end{array}\right\} \Rightarrow \left.\begin{array}{r}(\exists W \in O(Y, f(x)))(cl(W) \subseteq V)\\ f \text{ is contra } e^*\theta\text{-continuous}\end{array}\right\} \Rightarrow$$

$$\Rightarrow (\exists U \in e^*\theta O(X, x))(f[U] \subseteq cl(W) \subseteq V). \qquad \square$$

**Theorem 3.4.** *Let $\{X_\alpha | \alpha \in \Lambda\}$ be any family of topological spaces. If a function $f : X \to \Pi X_\alpha$ is a contra $e^*\theta$-continuous function, then $Pr_\alpha \circ f : X \to X_\alpha$ is contra $e^*\theta$-continuous for each $\alpha \in \Lambda$, where $Pr_\alpha$ is the projection of $\Pi X_\alpha$ onto $X_\alpha$.*

*Proof.* Let $\alpha \in \Lambda$ and $U_\alpha \in RO(X_\alpha)$.

$$\left.\begin{array}{r}\alpha \in \Lambda \Rightarrow Pr_\alpha \text{ is continuous}\\ U_\alpha \in O(X_\alpha)\end{array}\right\} \Rightarrow \left.\begin{array}{r}Pr_\alpha^{-1}[U_\alpha] \in O(\Pi X_\alpha)\\ f \text{ is c.}e^*\theta\text{.c.}\end{array}\right\} \Rightarrow$$

$$\Rightarrow (Pr_\alpha \circ f)^{-1}[U_\alpha] = f^{-1}[Pr_\alpha^{-1}[U_\alpha]] \in e^*\theta C(X). \qquad \square$$

**Definition 3.3.** A function $f : X \to Y$ is called weakly $e^*$-irresolute [20] (resp. strongly $e^*$-irresolute [20]) if $f^{-1}[A]$ is $e^*$-$\theta$-open in $X$ (resp. $e^*$-$\theta$-open) for every $e^*$-$\theta$-open (resp. $e^*$-open) set $A$ of $Y$.

**Theorem 3.5.** *Let $f : X \to Y$ and $g : Y \to Z$ and $g \circ f : X \to Z$ functions. Then the following properties hold:*

*(1) If $f$ is contra $e^*\theta$-continuous and $g$ is continuous, then $g \circ f$ is contra $e^*\theta$-continuous.*

*(2) If $f$ is $e^*\theta$-continuous and $g$ is contra-continuous, then $g \circ f$ is contra $e^*\theta$-continuous.*

*(3) If $f$ is contra $e^*\theta$-continuous and $g$ is contra-continuous, then $g \circ f$ is $e^*\theta$-continuous.*

*(4) If $f$ is weakly $e^*$-irresolute and $g$ is contra $e^*\theta$-continuous, then $g \circ f$ is contra $e^*\theta$-continuous.*

*(5) If $f$ is strongly $e^*$-irresolute and $g$ is contra $e^*$-continuous, then $g \circ f$ is contra $e^*\theta$-continuous.*

*Proof.* Straightforward. $\square$

# 4 Some fundamental properties of contra $e^*\theta$-continuous functions

**Definition 4.1.** A topological space $X$ is said to be:

a) $e^*\theta$-$T_0$ [3] if for any distinct pair of points $x$ and $y$ in $X$, there is an $e^*\theta$-open set $U$ in $X$ containing $x$ but not $y$ or an $e^*\theta$-open set $V$ in $X$ containing $y$ but not $x$.

b) $e^*\theta$-$T_1$ [3] if for any distinct pair of points $x$ and $y$ in $X$, there is an $e^*\theta$-open set $U$ in $X$ containing $x$ but not $y$ and an $e^*\theta$-open set $V$ in $X$ containing $y$ but not $x$.

c) $e^*\theta$-$T_2$ [3] (resp. $e^*$-$T_2$ [10]) if for every pair of distinct points $x$ and $y$, there exist two $e^*\theta$-open (resp. $e^*$-open) sets $U$ and $V$ such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

**Lemma 4.1.** *[3] For a topological space $X$, the following properties are equivalent:*

*(1) $(X, \tau)$ is $e^*\theta$-$T_0$.*

*(2) $(X, \tau)$ is $e^*\theta$-$T_1$.*

*(3) $(X, \tau)$ is $e^*\theta$-$T_2$.*

*(4) $(X, \tau)$ is $e^*$-$T_2$.*

*(5) For every pair of distinct points $x, y \in X$, there exist $U \in e^*O(X, x)$ and $V \in e^*O(X, y)$ such that $e^*$-$cl(U) \cap e^*$-$cl(V) = \emptyset$.*

*(6) For every pair of distinct points $x, y \in X$, there exist $U \in e^*R(X, x)$ and $V \in e^*R(X, y)$ such that $U \cap V = \emptyset$.*

*(7) For every pair of distinct points $x, y \in X$, there exist $U \in e^*\theta O(X, x)$ and $V \in e^*\theta O(X, y)$ such that $e^*$-$cl_\theta(U) \cap e^*$-$cl_\theta(V) = \emptyset$.*

**Theorem 4.1.** *A topological space $X$ is $e^*\theta$-$T_2$ if and only if the singletons are $e^*$-$\theta$-closed sets.*

*Proof.* Necessity. Let $x \in X$ and $X$ is $e^*\theta$-$T_2$.

$$\left.\begin{array}{r} y \notin \{x\} \Rightarrow x \neq y \\ X \text{ is } e^*\theta\text{-}T_2 \end{array}\right\} \Rightarrow (\exists U_y \in e^*\theta O(X, y))(\exists V_y \in e^*\theta O(X, x))(U_y \cap V_y = \emptyset)$$

$$\left.\begin{array}{r} \Rightarrow (\exists U_y \in e^*\theta O(X, y))(x \notin U_y) \\ \mathcal{A} := \{U_y | y \notin \{x\} \Rightarrow (\exists U_y \in e^*\theta O(X, y))(x \notin U_y)\} \subseteq e^*\theta O(X) \end{array}\right\} \Rightarrow$$

$$\Rightarrow X \setminus \{x\} = \bigcup \mathcal{A} \in e^*\theta O(X) \Rightarrow \{x\} \in e^*\theta C(X).$$

Sufficiency. Suppose that $\{x\}$ is $e^*$-$\theta$-closed for every $x \in X$. Let $x, y \in X$ with $x \neq y$.

$$\left.\begin{array}{r} x \neq y \Rightarrow y \in X \setminus \{x\} \\ x \in X \Rightarrow \{x\} \in e^*\theta C(X) \end{array}\right\} \Rightarrow X \setminus \{x\} \in e^*\theta O(X, y).$$

Then $X$ is $e^*\theta$-$T_0$. On the other hand, the notions of $e^*\theta$-$T_0$ and $e^*\theta$-$T_1$ are equivalent from Lemma 4.1. Thus $X$ is $e^*\theta$-$T_1$. □

**Theorem 4.2.** *If $f$ is a contra $e^*\theta$-continuous injection of a topological space $X$ into a Urysohn space $Y$, then $X$ is $e^*\theta$-$T_2$.*

*Proof.* Let $x_1, x_2 \in X$ and $x_1 \neq x_2$.

$$\left.\begin{array}{r} x_1 \neq x_2 \\ f \text{ is injective} \end{array}\right\} \Rightarrow \left.\begin{array}{r} f(x_1) \neq f(x_2) \\ Y \text{ is Urysohn} \end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{r}\Rightarrow (\exists U\in O(Y,y_1))(\exists V\in O(Y,y_2))(cl(U)\cap cl(V)=\emptyset)\\ f \text{ is c.}e^*\theta\text{.c. at } x_1 \text{ and } x_2\end{array}\right\}\Rightarrow$$

$$\Rightarrow (\exists A\in e^*\theta O(X,x_1))(\exists B\in e^*\theta O(X,x_2))(f[A]\cap f[B]\subseteq cl(U)\cap cl(V)=\emptyset)$$

$$\Rightarrow (\exists A\in e^*\theta O(X,x_1))(\exists B\in e^*\theta O(X,x_2))(A\cap B=\emptyset). \qquad \square$$

**Definition 4.2.** A topological space $X$ is said to be:

a) Weakly Hausdorff [21] (briefly weakly-$T_2$) if every point of $X$ is an intersection of regularly closed sets of $X$.

b) Ultra Hausdorff [22] if for each pair of distinct points $x$ and $y$ in $X$, there exist clopen sets $U$ and $V$ containing $x$ and $y$, respectively such that $U\cap V=\emptyset$.

**Theorem 4.3.** *Let $f:X\to Y$ be a function. Then the following properties are hold:*

*(1) If $f$ is a contra $e^*\theta$-continuous injection and $Y$ is $T_0$, then $X$ is $e^*\theta$-$T_2$.*

*(2) If $f$ is a contra $e^*\theta$-continuous injection and $Y$ is Ultra Hausdorff, then $X$ is $e^*\theta$-$T_2$.*

*Proof.* *(1)* Let $x_1,x_2\in X$ and $x_1\neq x_2$.

$$\left.\begin{array}{r}\left.\begin{array}{r}(x_1,x_2\in X)(x_1\neq x_2)\\ f \text{ is injective}\end{array}\right\}\Rightarrow f(x_1)\neq f(x_2)\\ Y \text{ is } T_0\end{array}\right\}\Rightarrow$$

$$\Rightarrow [(\exists V\in O(Y,f(x_1)))(f(x_2)\in V)\vee(\exists U\in O(Y,f(x_2)))(f(x_1)\in U)]$$

$$\left.\begin{array}{r}\Rightarrow (f(x_1)\notin Y\setminus V)(Y\setminus V\in C(Y,f(x_2)))\\ f \text{ is c.}e^*\theta\text{.c.}\end{array}\right\}\Rightarrow x_1\notin f^{-1}[Y\setminus V]\in e^*\theta O(X,x_2).$$

Therefore $X$ is $e^*\theta$-$T_0$ and by Theorem 4.1 $X$ is $e^*\theta$-$T_2$.

*(2)* It is not difficult to see that this item is immediate consequence of *(1)* by Lemma 4.1. $\square$

**Definition 4.3.** A space $X$ is said to be:

a) $e^*\theta$-connected if $X$ cannot be expressed as the disjoint union of two non-empty $e^*$-$\theta$-open sets.

b) $e^*\theta$-normal if for each pair of non-empty disjoint closed sets can be separated by disjoint $e^*$-$\theta$-open sets.

**Theorem 4.4.** *If $f:X\to Y$ is a contra $e^*\theta$-continuous surjection and $X$ is $e^*\theta$-connected, then $Y$ is connected.*

*Proof.* Suppose that $Y$ is not connected.

$$Y \text{ is not connected} \Rightarrow (\exists U_1,U_2\in O(Y)\setminus\{\emptyset\})(U_1\cap U_2=\emptyset)(U_1\cup U_2=Y)$$

$$\left.\begin{array}{r}\Rightarrow U_1,U_2\in CO(Y)\\ f \text{ is c.}e^*\theta\text{.c. surjection}\end{array}\right\}\Rightarrow$$

$$\Rightarrow (f^{-1}[U_1],f^{-1}[U_2]\in e^*\theta O(X)\setminus\{\emptyset\})(f^{-1}[U_1]\cap f^{-1}[U_2]=\emptyset)(f^{-1}[U_1]\cup f^{-1}[U_2]=X).$$

This is a contradiction to the fact that $X$ is $e^*\theta$-connected. $\square$

**Theorem 4.5.** *If $f : X \to Y$ is a contra $e^*\theta$-continuous closed injection and $Y$ is normal, then $X$ is $e^*\theta$-normal.*

*Proof.* Let $F_1, F_2 \in C(X)$ and $F_1 \cap F_2 = \emptyset$.

$$\left.\begin{array}{r}(F_1, F_2 \in C(X))(F_1 \cap F_2 = \emptyset)\\ f \text{ is closed injection}\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{r}(f[F_1], f[F_2] \in C(Y))(f[F_1 \cap F_2] = f[F_1] \cap f[F_2] = \emptyset)\\ Y \text{ is normal}\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{r}(\exists V_1, V_2 \in O(Y))(f[F_1] \subseteq V_1)(f[F_2] \subseteq V_2)(V_1 \cap V_2 = \emptyset)\\ Y \text{ is normal}\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{r}(\exists G_1, G_2 \in O(Y))(f[F_1] \subseteq G_1 \subseteq cl(G_1) \subseteq V_1)(f[F_2] \subseteq G_2 \subseteq cl(G_2) \subseteq V_2)(V_1 \cap V_2 = \emptyset)\\ f \text{ is c.}e^*\theta.c.\end{array}\right\} \Rightarrow$$

$$\Rightarrow (f^{-1}[cl(G_1)], f^{-1}[cl(G_2)] \in e^*\theta O(X))(F_1 \subseteq f^{-1}[cl(G_1)])(F_2 \subseteq f^{-1}[cl(G_2)])$$
$$(f^{-1}[cl(G_1)] \cap f^{-1}[cl(G_2)] = \emptyset).$$ □

**Definition 4.4.** A function $f : X \to Y$ has a contra $e^*\theta$-closed graph if for each $(x, y) \notin G(f)$, there exist $U \in e^*\theta O(X, x)$ and $V \in C(Y, y)$ such that $(U \times V) \cap G(f) = \emptyset$.

**Lemma 4.2.** *The graph $G(f)$ of a function $f : X \to Y$ is contra $e^*\theta$-closed in $X \times Y$ if and only if for each $(x, y) \notin G(f)$, there exist $U \in e^*\theta O(X, x)$ and $V \in C(Y, y)$ such that $f[U] \cap V = \emptyset$.*

*Proof.* Straightforward. □

**Theorem 4.6.** *If $f : X \to Y$ is contra $e^*\theta$-continuous and $Y$ is Urysohn, then $f$ has a contra $e^*\theta$-closed graph.*

*Proof.* Let $(x, y) \notin G(f)$.

$$\left.\begin{array}{r}(x, y) \notin G(f) \Rightarrow y \neq f(x)\\ Y \text{ is Urysohn}\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{r}(\exists V \in O(Y, f(x)))(\exists W \in O(Y, y))(cl(V) \cap cl(W) = \emptyset)\\ f \text{ is c.}e^*\theta.c.\end{array}\right\} \Rightarrow$$

$$\Rightarrow (\exists U \in e^*\theta O(X, x))(f[U] \subseteq cl(V))(cl(V) \cap cl(W) = \emptyset)$$

$$\Rightarrow (\exists U \in e^*\theta O(X, x))(f[U] \cap W \subseteq f[U] \cap cl(W) = \emptyset).$$ □

**Theorem 4.7.** *Let $f : X \to Y$ be a function and $g : X \to X \times Y$ the graph function of $f$, defined by $g(x) = (x, f(x))$ for every $x \in X$. If $g$ is contra $e^*\theta$-continuous, then $f$ is contra $e^*\theta$-continuous.*

*Proof.* Let $V \in O(Y)$.

$$\left.\begin{array}{r}V \in O(Y) \Rightarrow X \times V \in O(X \times Y)\\ g \text{ is c.}e^*\theta\text{.c.}\end{array}\right\} \Rightarrow f^{-1}[V] = g^{-1}[X \times V] \in e^*\theta C(X).$$ □

**Theorem 4.8.** *If $f : X \to Y$ has a contra $e^*\theta$-closed graph and injective, then $X$ is $e^*\theta$-$T_1$.*

*Proof.* Let $x_1, x_2 \in X$ and $x_1 \neq x_2$.

$$\left.\begin{array}{r} \left.\begin{array}{r}(x_1, x_2 \in X)(x_1 \neq x_2) \\ f \text{ is injective}\end{array}\right\} \Rightarrow f(x_1) \neq f(x_2) \Rightarrow (x_1, f(x_2)) \notin G(f) \\ G(f) \text{ is contra } e^*\theta\text{-closed}\end{array}\right\} \Rightarrow$$

$$\Rightarrow (\exists U \in e^*\theta O(X, x_1))(\exists V \in O(Y, f(x_2)))(f[U] \cap V = \emptyset)$$
$$\Rightarrow (\exists U \in e^*\theta O(X, x_1))(\exists V \in O(Y, f(x_2)))(U \cap f^{-1}[V] = \emptyset)$$
$$\Rightarrow (\exists U \in e^*\theta O(X, x_1))(x_2 \notin U)$$

Then $X$ is $e^*\theta$-$T_0$. On the other hand, the notions of $e^*\theta$-$T_0$ and $e^*\theta$-$T_1$ are equivalent from Lemma 4.1. Thus $X$ is $e^*\theta$-$T_1$. □

**Definition 4.5.** A topological space $X$ is said to be:

a) Strongly $S$-closed [6] if every closed cover of $X$ has a finite subcover.

b) Strongly $e^*\theta C$-compact [3] if every $e^*$-$\theta$-closed cover of $X$ has a finite subcover.

c) $e^*\theta$-compact if every $e^*$-$\theta$-open cover of $X$ has a finite subcover.

d) $e^*\theta$-space if every $e^*$-$\theta$-closed set is closed.

**Theorem 4.9.** *If $f : X \to Y$ has a contra $e^*\theta$-closed graph and $X$ is an $e^*\theta$-space, then $f^{-1}[K]$ is closed in $X$ for every strongly $S$-closed subset $K$ of $Y$.*

*Proof.* Let $K$ is strongly $S$-closed in $Y$ and let $x \notin f^{-1}[K]$.

$$\left.\begin{array}{r} x \notin f^{-1}[K] \Rightarrow f(x) \notin K \Rightarrow (\forall y \in K)(y \neq f(x)) \Rightarrow (x, y) \notin G(f) \\ G(f) \text{ is } e^*\theta\text{-closed}\end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{r} \Rightarrow (\exists U_y \in e^*\theta O(X, x))(\exists V_y \in C(Y, y))(f[U_y] \cap V_y = \emptyset) \\ \mathcal{A} := \{K \cap V_y | y \in K\}\end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{r} \left.\begin{array}{r}\Rightarrow (\mathcal{A} \subseteq C(Y))(K = \bigcup \mathcal{A}) \\ K \text{ is strongly } S\text{-closed in } Y\end{array}\right\} \Rightarrow (\exists \mathcal{A}^* \subseteq \mathcal{A})(|\mathcal{A}^*| < \aleph_0)(K \subseteq \bigcup \mathcal{A}^*) \\ U := \cap\{U_y | U_y \in \mathcal{A}^*\}\end{array}\right\} \overset{X \text{ is } e^*\theta\text{-space}}{\Rightarrow}$$

$$\Rightarrow (U \in O(X, x))(f[U] \cap K = \emptyset) \Rightarrow (U \in O(X, x))(U \cap f^{-1}[K] = \emptyset) \Rightarrow$$
$$\Rightarrow (U \in O(X, x))(U \subseteq \setminus f^{-1}[K]) \Rightarrow x \in int(X \setminus f^{-1}[K]) \Rightarrow x \in X \setminus cl(f^{-1}[K]) \Rightarrow x \notin cl(f^{-1}[K]).$$

□

**Theorem 4.10.** *If $f : X \to Y$ is a contra $e^*\theta$-continuous surjection and $X$ is strongly $e^*\theta C$-compact, then $Y$ is compact.*

*Proof.* Let $\mathcal{B} \subseteq O(Y)$ and $Y = \bigcup \mathcal{B}$.

$$\left.\begin{array}{r} \left.\begin{array}{r}(\mathcal{B} \subseteq O(Y))(Y = \bigcup \mathcal{B}) \\ f \text{ is c.}e^*\theta\text{.c.}\end{array}\right\} \Rightarrow (\mathcal{A} := \{f^{-1}[B] | B \in \mathcal{B}\} \subseteq e^*\theta C(X))(X = \bigcup \mathcal{A}) \\ X \text{ is strongly } e^*\theta C\text{-compact}\end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{r} \Rightarrow (\exists \mathcal{A}^* \subseteq \mathcal{A})(|\mathcal{A}^*| < \aleph_0)(X = \bigcup \mathcal{A}^*) \\ f \text{ is surjective}\end{array}\right\} \Rightarrow (\mathcal{B}^* := \{f[A] | A \in \mathcal{A}^*\} \subseteq \mathcal{B})(|\mathcal{B}^*| < \aleph_0)(Y = \bigcup \mathcal{B}^*).$$

□

**Theorem 4.11.** *Let $f : X \to Y$ be a function. Then the following properties are hold:*

*(1) If $f$ is a contra $e^*\theta$-continuous surjection and $X$ is $e^*\theta$-compact, then $Y$ is strongly $S$-closed.*

*(2) If $f$ is a contra $e^*\theta$-continuous surjection and $X$ is $e^*\theta$-compact and $e^*\theta$-space, then $Y$ is strongly $e^*\theta C$-compact.*

*Proof.* *(1)* Let $\mathcal{B} \subseteq C(Y)$ and $Y = \bigcup \mathcal{B}$.

$$\left.\begin{array}{r}(\mathcal{B} \subseteq C(Y))(Y = \bigcup \mathcal{B})\\ f \text{ is c.}e^*\theta\text{.c.}\end{array}\right\} \Rightarrow \left.\begin{array}{r}(\mathcal{A} := \{f^{-1}[B] | B \in \mathcal{B}\} \subseteq e^*\theta O(X))(X = \bigcup \mathcal{A})\\ X \text{ is } e^*\theta\text{-compact}\end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{r}\Rightarrow (\exists \mathcal{A}^* \subseteq \mathcal{A})(|\mathcal{A}^*| < \aleph_0)(X = \bigcup \mathcal{A}^*)\\ f \text{ is surjective}\end{array}\right\}$$

$$\Rightarrow (\mathcal{B}^* := \{f[A] | A \in \mathcal{A}^*\} \subseteq \mathcal{B})(|\mathcal{B}^*| < \aleph_0)(Y = \bigcup \mathcal{B}^*)$$

*(2)* Let $\mathcal{B} \subseteq e^*\theta C(Y)$ and $Y = \bigcup \mathcal{B}$.

$$\left.\begin{array}{r}(\mathcal{B} \subseteq e^*\theta C(Y))(Y = \bigcup \mathcal{B})\\ X \text{ is } e^*\theta\text{-space}\end{array}\right\} \Rightarrow \left.\begin{array}{r}(\mathcal{B} \subseteq C(Y))(\bigcup \mathcal{B} = Y)\\ f \text{ is c.}e^*\theta\text{.c.}\end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{r}\Rightarrow (\exists \mathcal{A}^* \subseteq \mathcal{A})(|\mathcal{A}^*| < \aleph_0)(X = \bigcup \mathcal{A}^*)\\ f \text{ is surjective}\end{array}\right\} \Rightarrow (\mathcal{B}^* := \{f[A] | A \in \mathcal{A}^*\} \subseteq \mathcal{B})(|\mathcal{B}^*| < \aleph_0)(Y = \bigcup \mathcal{B}^*).$$

□

**Theorem 4.12.** *If $f : X \to Y$ is a weakly $e^*$-irresolute surjection and $X$ is strongly $e^*\theta C$-compact, then $Y$ is strongly $e^*\theta C$-compact.*

*Proof.* Let $\mathcal{B} \subseteq e^*\theta C(Y)$ and $Y = \bigcup \mathcal{B}$.

$$\left.\begin{array}{r}(\mathcal{B} \subseteq e^*\theta C(Y))(Y = \bigcup \mathcal{B})\\ f \text{ is weakly } e^*\text{-irresolute}\end{array}\right\} \Rightarrow \left.\begin{array}{r}(\mathcal{A} := \{f^{-1}[B] | B \in \mathcal{B}\} \subseteq e^*\theta C(X))(X = \bigcup \mathcal{A})\\ X \text{ is strongly } e^*\theta C\text{-compact}\end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{r}\Rightarrow (\exists \mathcal{A}^* \subseteq \mathcal{A})(|\mathcal{A}^*| < \aleph_0)(X = \bigcup \mathcal{A}^*)\\ f \text{ is surjective}\end{array}\right\} \Rightarrow$$

$$\Rightarrow (\mathcal{B}^* := \{f[A] | A \in \mathcal{A}^*\} \subseteq \mathcal{B})(|\mathcal{B}^*| < \aleph_0)(Y = \bigcup \mathcal{B}^*). \qquad \square$$

We recall that the product space $X = X_1 \times \ldots \times X_n$ has property $P_{e^*\theta}$ [3] if $A_i$ is an $e^*\theta$-open set in a topological space $X_i$ for $i = 1, 2, \ldots n$, then $A_1 \times \ldots \times A_n$ is also $e^*\theta$-open in the product space $X = X_1 \times \ldots \times X_n$.

**Theorem 4.13.** *Let $f : X_1 \to Y$ and $g : X_2 \to Y$ be two functions, where*

*(i) $X = X_1 \times X_2$ has the property $P_{e^*\theta}$,*

*(ii) $Y$ is a Urysohn space,*

*(iii) $f$ and $g$ are contra $e^*\theta$-continuous,*

*then* $\{(x_1,x_2)|f(x_1)=g(x_2)\}$ *is* $e^*\theta$*-closed in the product space* $X=X_1\times X_2$*.*

*Proof.* Let $(x_1,x_2)\notin A:=\{(x_1,x_2)|f(x_1)=g(x_2)\}$.

$$\left.\begin{array}{r}(x_1,x_2)\notin A\Rightarrow f(x_1)\neq g(x_2)\\ Y\text{ is Urysohn}\end{array}\right\}\Rightarrow$$

$$\left.\begin{array}{r}\Rightarrow(\exists V_1\in O(Y,f(x_1)))(\exists V_2\in O(Y,g(x_2)))(cl(V_1)\cap cl(V_2)=\emptyset)(cl(V_1),cl(V_2)\in RC(Y))\\ f\text{ and }g\text{ are c.}e^*\theta\text{.c.}\end{array}\right\}\Rightarrow$$

$$\left.\begin{array}{r}\Rightarrow(f^{-1}[cl(V_1)]\in e^*\theta O(X_1,x_1))(g^{-1}[cl(V_2)]\in e^*\theta O(X_2,x_2))\\ X=X_1\times X_2\text{ has the Property }P_{e^*\theta}\end{array}\right\}\Rightarrow$$

$$\Rightarrow((x_1,x_2)\in f^{-1}[cl(V_1)]\times g^{-1}[cl(V_2)]\in e^*\theta O(X_1\times X_2))(f^{-1}[cl(V_1)]\times g^{-1}[cl(V_2)]\subseteq\setminus A)\Rightarrow$$

$$\Rightarrow\setminus A\in e^*\theta O(X_1\times X_2)\Rightarrow A\in e^*\theta C(X_1\times X_2).$$

□

# 5 Acknowledgements

This work is supported by the Scientific Research Proyect Fund of Muğla Sitki Koçman University under the project number 17/277.

# References


[1] Abd El-Monsef, M. E.; El-Deeb, S. N. and Mahmoud, R. A. *$\beta$-open sets and $\beta$-continuous mappings*, Bull. Fac. Sci., Assiut Univ. **12** (1983), 77–90.

[2] Andrijević, D. *On b-open sets*, Mat. Vesnik **48** (1996), 59–64.

[3] Ayhan, B.S. and Özkoç, M. *On almost contra $e^*\theta$-continuous functions*. Jordan Journal of Mathematics and Statistics, **11**(4) (2018), 383–408.

[4] Caldas, M. *On Contra $\beta\theta$-Continuous Functions*, Proyecciones J. Math., **32**(4) (2013), 333–346.

[5] Caldas, M. and Jafari, S. *Some properties of contra-$\beta$-continuous functions*, Mem. Fac. Sci. Koch Univ. (Math) **22** (2001), 19–28.

[6] Dontchev, J. *Contra-continuous functions and strongly $S$-closed spaces*, Internat. J. Math. Math. Sci., **19** (1996), 303–310.

[7] Dontchev, J. and Noiri, T. *Contra-semicontinuous functions*, Math. Pannonica **10** (1999), 159–168.

[8] Ekici, E. *On e-open sets, $\mathcal{DP}^*$-sets and $\mathcal{DPE}^*$-sets and decompositions of continuity*, Arabian J. Sci. Eng. **33**(2A) (2008), 269–282.

[9] Ekici, E. *On $e^*$-open sets and $(\mathcal{D},\mathcal{S})^*$-sets*, Math. Morav. **13**(1) (2009), 29–36.

[10] Ekici, E. *New forms of contra-continuity*, Carpathian J. Math. **24**(1) (2008), 37–45.

[11] Farhan, A. M. and Yang, X. S. *New types of strongly continuous functions in topological spaces via $\delta$-$\beta$-open sets*, Eur. J. Pure Appl. Math., **8**(2) (2015), 185–200.

[12] Jafari, S. and Noiri, T. *Contra-$\alpha$-continuous functions between topological spaces*, Iran. Int. J. Sci., **2** (2001), 153–167.

[13] Jafari, S. and Noiri, T. *On contra precontinuous functions*, Bull. Malaysian Math. Sci. Soc., **25** (2002), 115–128.

[14] Levine, N. *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly **70** (1963),36–41.

[15] Mashhour, A. S.; Abd El-Monsef, M. E. and El-Deeb, S. N. *On precontinuous and weak precontinuous mappings*, Proc. Math. Phys. Soc. Egypt **53** (1982), 47–53.

[16] Mrsevic, M. *On pairwise $R_0$ and pairwise $R_1$ bitopological spaces*, Bull. Math. Soc. Sci. Math RS Roumano, (N.S.) **30**(78) (1986), 141–148.

[17] Nasef, A. A. *Some properties of contra-$\gamma$-continuous functions*, Chaos Solitons and Fractals **24** (2005), 471–477.

[18] Njastad, O. *On some classes of nearly open sets*, Pacific J. Math. **15** (1965), 961–970.

[19] Noiri, T. *Weak and strong forms of $\beta$-irresolute functions*, Acta Math. Hungar., **99** (2003), 315–328.

[20] Özkoç, M. and Atasever, K. S. *On some forms of $e^*$-irresoluteness* (Accepted in JLTA).

[21] Soundararajan, T. *Weakly Hausdorff space and the cardinality of topological spaces*, General Topology and its Relation to Modern Analysis and Algebra III, Proc. Conf. Kampur, 168, Acad. Prague (1971), 301–306.

[22] Staum, R. *The algebra of bounded continuous functions into a nonarchimedean field*, Pacific J. Math., **50** (1974), 169–85.

[23] Stone, M. H. *Applications of the theory of Boolean rings to general topology*, Trans. Amer. Math. Soc. **41** (1937), 375–381.

[24] Veličko, N. V. *H-closed topological spaces*, Amer. Math. Soc. Transl. (2) **78** (1968), 103–118.